Mechanical Engineering

Computer Graphics and Image Processing

1. You are writing code for a new graphics card that is software programmable, rather than having the algorithms embedded in hardware. You want to write a fast triangle-drawing program to test the card.
    i. Provide pseudocode, or similar, that draws a triangle with a constant color. Assume that the inputs are the color of the triangle and three two-dimensional points, representing the three vertices of the triangle. Further, assume that all three vertices lie on the visible screen and that no anti-aliasing is required. You may assume that there is a function to set a pixel, (x,y), to a particular color, c, e.g. setpixel (x,y,c), but you must provide pseudocode for any other functions that you require. Your answer should be sufficiently detailed that it could be transferred directly into a language such as Java but your answer does not, itself, have to be syntactically correct. [13 marks]
    ii. Outline the extra steps required to draw a triangle specified by three dimensional points in world space, where the triangle may extend beyond the edges of the screen after conversion to screen space. [4 marks]
    iii. Outline the steps required to calculate the triangle's color, assuming diffuse shading, with multiple point lights, but still producing a single color for the whole triangle. [3 marks]
2. A new computer display is being designed to project large images onto the walls of buildings. It works by reflecting a laser beam off a mirror mounted on a pair of loudspeaker coils. Two analogue signals control the horizontal and vertical deflection of the laser beam by driving the coils. A third digital signal turns the laser on and off. The deflection coils can be driven up to a frequency of 20 kHz, and the laser can be turned on and off at a frequency of 10 MHz. Two approaches are being considered: raster scan and calligraphic.
    i. What resolution two-level (on/off) raster display could the hardware support? [4 marks]
    ii. How might grey levels be provided? What resolution raster display could the

hardware support if eight levels of grey were required? [3 marks]

iii. An alternative approach would be a random scan calligraphic display that drew lines by deflecting the mirrors to the beginning of a line, turning the laser on, deflecting the mirrors to the end of the line (ensuring that the intermediate deflections lay along a straight line), and turning the laser off again. How many lines could be drawn on this calligraphic display before the image flickered? [2 marks]

iv. The analogue signals controlling the horizontal and vertical deflection coils are produced by fast digital-to-analogue converters with 10-bit inputs. Present an algorithm for generating intermediate coordinates along the line to ensure that lines drawn on the calligraphic display are straight and also evenly illuminated. [7 marks]

v. How could the algorithm be adapted to draw arcs of circles? [2 marks]

vi. What additional hardware might be useful to assist with generating the intermediate coordinates from the end points of lines? [2 marks]

3. What are the main criteria to be considered in the design of a line drawing algorithm for a raster graphics display? [2 marks]
4. Describe an algorithm to fill a series of pixels running from (x0,y0) to (xi,yi) that meets these criteria, explaining why it does so. Answers should consist of more than a fragment of pseudo-code. [6 marks]
5. A new volumetric display stores an image as a three-dimensional array of volume elements or voxels. Reformulate the design and implementation of the line-drawing algorithm to fill a series of voxels running from (x0, y0, z0) to (x1, y1, z1). [6 marks]
6. How would this line-drawing algorithm be used to draw Bezier curves in three dimensions? [6 marks]
7. Describe how transform coding can be used to compress image data. [4 marks]
8. Explain the Walsh-Hadamard transform on a one-dimensional array of 4 greyscale values. [4 marks]
9. Extend this to the Walsh transform for a two-dimensional array of 2 x 2 greyscale values. [4 marks]

10. Develop an analogous transform for a three-dimensional array of 2 x 2 x 2 greyscale values. [8marks]

    Include sufficient algebra in your answers to allow a competent programmer to implement the algorithm.

11. Explain the difference between an explicit formula, a closed form and a parametric form for a curve in two dimensions. Give examples to illustrate your answer. [3 marks]

12. Explain the term mathematical continuity (Cn) when joining two curves. [2 marks]

13. Give the formulation of a cubic Bezier curve in two dimensions, explaining the role of the parameter and control points. [4 marks]

14. Consider the joint between two cubic Bezier curves. State and prove constraints on their control points to ensure:

    i. Co continuity at the joint; [2 marks]

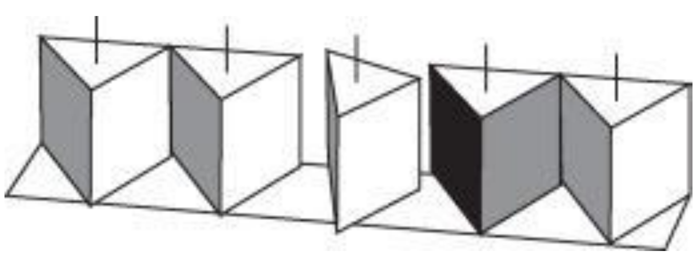
    ii. Ci continuity at the joint; [4 marks]

    iii. C2 continuity at the joint. [3 marks]

15. Discuss the implications of requiring C3 continuity at the joint between two cubic Bezier curves. [2 marks]

16. Describe, in detail, an algorithm to perform error diffusion on a greyscale image. Your algorithm should take a greyscale image, with eight bits per pixel, and convert it to a black and white image, with one bit per pixel, at the same resolution. [8 marks]

17. An inventor produces a display where each pixel can have one of three values: white, mid-grey, or black. Such a display can be built by, for example, using rotating triangular blocks of painted wood. The figure shows the back view of a row of five pixels with the central pixel turning. From left to right the pixels are showing, to the front side: black, black, turning, white, black.

    Modify your algorithm in part (a) to handle these three-valued pixels. [4 marks]

18. Describe, in detail, the modifications required to turn the display described above into a

color display. Your display, through use of an appropriate error diffusion algorithm, should be able to display error-diffused versions of 24-bit RGB color images.
[8 marks]

19. Given a sequence of points (U)™=0 on a plane, consider the problem of interpolating a smooth curve through all of the points in order by constructing a sequence of polynomial parametric functions, one for each interval [Vi, V+^h—o.
    i. What is meant by Ck continuity at the junction between two curve segments? [2 marks]
    ii. Explain how the degree of the polynomial function for a curve segment constrains the continuity at its two ends. What continuity can be achieved at each end of a cubic segment? [4 marks]
    iii. Derive a cubic parametric function for the interval [Vi, Vi+1] where 0 < i < n — 1. [10 marks]
    iv. What special provision would have to be made for the segments [V0, V1] and [V n-i, Vn]? [4 marks]
20. Given a model of a scene represented as a set of triangles in three-dimensional space defining its surfaces, consider the problem of rendering it on a raster display. Write brief notes on:
    i. the data that would be stored for each triangle; [2 marks]
    ii. perspective projection from an arbitrary viewpoint; [5 marks]
    iii. clipping the data to a suitable viewing frustrum; [5 marks]
    iv. identifying pixels on the screen within a triangle; [3 marks]
    v. resolving hidden surfaces using a Z-buffer. [5 marks]
21. Consider the transformations used in the construction and rendering of a three-dimensional model on a screen.
    a. List the three principal transformations in the processing pipeline and explain their roles. [6 marks]
    b. Why is it convenient to represent the transformations as matrices? [2 marks]
    c. What are homogeneous coordinates? Explain how they are used in modelling these transformations as matrices. [2 marks]
    d. Derive the matrix to represent a perspective transformation for a viewer at the

origin of a point in three dimensions to a point on a screen in the plane z = d.
[5 marks]

e. Perspective in classical art has vanishing points towards which parallel lines converge. Explain mathematically why this is the case and show how to calculate the location on the screen of the vanishing point for lines in a particular direction.
[5 marks]

[Hint: It may be helpful to represent lines parametrically in vector form as P(s) = A + sV where V is a direction and A is any point on the line.]